Available online at www.sciencedirect.com
SCIENCE DIRECT
Journal of Sound and Vibration 267 (2003) 335–354
JOURNAL OF SOUND AND VIBRATION
www.elsevier.com/locate/jsvi

Stochastic control of flexible beam in random flutter

Hoon Heo[a,*], Yun Hyun Cho[b], Dae Jung Kim[c]

[a] *Department of Control & Instrumentation Engineering, Korea University,1-5ka, Anam-Dong Sungbuk-ku, Seoul 136-072, South Korea*
[b] *Department of Control & Instrumentation Engineering, Korea University, Seoul 136-072, South Korea*
[c] *Graduate Student, Currently Research Engineer in Bosch Korea, 298 Bojungri, Kusungeup, Yongin, Kyungido 449-913, South Korea*

Received 21 July 1999; accepted 27 January 2003

Abstract

A new stochastic controller for a dynamic system under irregular disturbance has been developed and investigated via Monte-Carlo simulation and physical experiment. In order to design what we called a "Heo stochastic controller", the system equation is transformed to stochastic domain by F–P–K approach from physical domain. A "Heo stochastic controller" is designed in stochastic domain by using a conventional method such as a PI controller.

This paper consists of a basic description of F–P–K equation approach, the design of the "Heo stochastic controller", realization of the technique and its performance, and simulation results. A thin beam is adopted as an airfoil model, and then the newly designed "Heo stochastic controller" is implemented to the system. A flutter control simulation for a thin airfoil exposed to turbulent flow is conducted numerically and experimentally as well. The newly proposed "Heo-PI stochastic controller" shows promising performance.

1. Introduction

Dynamic systems are often exposed to various external disturbances in nature. Especially, random disturbance is the most common case. These include aerospace systems excited by atmospheric and boundary layer turbulence and jet noise; aircraft and vehicles subjected to track induced vibrations; ground based structures excited by earthquakes and wind; and offshore structures excited by wind and hydrodynamic wave-induced loads. In each case the physical variables exhibit random fluctuations in both space and time [1,4].

*Corresponding author. Tel.: +82-2-3290-3974; fax: +82-2-929-7808.
E-mail address: heo257@korea.ac.kr (H. Heo).

0022-460X/03/$ - see front matter © 2003 Elsevier Ltd. All rights reserved.
doi:10.1016/S0022-460X(03)00184-6

In recent years, dynamic systems have become larger and more complex. As a result, the random disturbance/noise effect on systems is receiving more attention in system design. Accordingly, its control in a more proper way draws a lot of attention. Many controller design techniques have been tried to reject or suppress random disturbances and have achieved reliable results. Some stochastic control techniques such as LQR, LQG, H_∞ etc., are effective at disturbance/noise rejection; these are designed in time domain or frequency domain. Nevertheless, these techniques don't use enough noise/disturbance information. LQR design does not use any random input signal information. LQG uses only correlation values on signal in estimation, such as Kalman filter and H_∞ filter. H_∞ controller uses noise maximum norm [2]. As an alternative controller design method for extended use of random signal and system information, a new concept for stochastic controller design has recently been proposed. Kim et al. (1995) showed the feasibility of stochastic controllers in stochastic observer design via the Fokker–Plank–Kolmogrov (F–P–K) approach also, Cho et al. (1998) developed the stochastic controller by using a GA based fuzzy controller in probabilistic domain. An experimental study for the control of base excited flexible beam model was carried on with piezo actuator and sensor in 1996. Wind tunnel models for realistic random disturbance have been used for experiments since 1997 [3,8,11].

2. Stochastic analysis

2.1. F–P–K procedure

The F–P–K method is one way of studying the behavior of a system probability density function subjected to random fluctuation externally, internally or interactively.

There are two basic assumptions for the derivation of F–P–K equation. First, random input is always sufficiently small, so that the perturbed motion can be determined by superimposing random fluctuations of first order smallness to a continuous mean trajectory. Second, the random process under consideration is a Markov process, and does not depend on its past history. The general form of the F–P–K equation with drift (increment of first order moment) and diffusion (increment of second order moment) coefficients is given as

$$\frac{\partial}{\partial t}p(\underline{X},t) = -\sum_{i=1}^{n}\frac{\partial}{\partial t}\{a_i(\underline{X},t)p(\underline{X},t)\} + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{\partial^2}{\partial X_i\partial X_j}\{b_{ij}(\underline{X},t)p(\underline{X},t)\}, \tag{1}$$

where drift coefficient, $a_i(\underline{X},t)$ and diffusion coefficient, $b_{ij}(\underline{X},t)$ are defined respectively as [5],

$$a_i(\underline{X},t) = \lim_{\Delta t\to 0}\frac{1}{\Delta t}E[x_i(t+\Delta t) - x_i(t)], \tag{2}$$

$$b_{ij}(\underline{X},t) = \lim_{\Delta t\to 0}\frac{1}{\Delta t}E[\{x_i(t+\Delta t) - x_i(t)\}\{x_j(t+\Delta t) - x_j(t)\}].$$

The solution of these equations gives the probabilistic behavior of the system response. In many cases, however, it is not possible to obtain a closed form analytical solution to the F–P–K equation of the dynamic system. Instead of seeking stationary or non-stationary solutions for the F–P–K equation, one can generate a set of differential equations for the response moments.

Let $\mathbf{\Phi}(\underline{X})$ be a general function of the response co-ordinate vector $\underline{X}$

$$\mathbf{\Phi}(\underline{X}) = X_1^{k_1} X_2^{k_2} \cdots X_n^{k_n} = \prod_{i=1}^{n} X_i^{k_i} \tag{3}$$

such that the following notation expresses the moments of order k_i

$$m_{k_1,k_2,\cdots,k_n} = E[\mathbf{\Phi}(\underline{X})] = \int \cdots \int_{-\infty}^{\infty} \mathbf{\Phi}(\underline{X}) p(\underline{X}, t)\, \mathrm{d}X_1\, \mathrm{d}X_2 \cdots \mathrm{d}X_n, \tag{4}$$

where $N = \sum_i^n k_i$

The differential equations of the response dynamic moments can be derived by multiplying both sides of the system F–P–K equation by $\mathbf{\Phi}(\underline{X})$ and integrating by parts over the entire state space $-\infty < \underline{X} < \infty$ [4,6].

$$\begin{aligned}
\dot{m}_{k_1,k_2,\cdots,k_n} &= \int \cdots \int_{-\infty}^{\infty} \mathbf{\Phi}(\underline{X}) \frac{\partial p(\underline{X}, t)}{\partial t} \mathrm{d}X_1\, \mathrm{d}X_2 \cdots \mathrm{d}X_n \\
&= -\int \cdots \int_{-\infty}^{\infty} \mathbf{\Phi}(\underline{X}) \sum_{i=1}^{n} \frac{\partial p(\underline{X}, t)}{\partial X_i} \{a_i(\underline{X}, t) p(\underline{X}, t)\}\, \mathrm{d}X_1\, \mathrm{d}X_2 \cdots \mathrm{d}X_n \\
&\quad + \frac{1}{2} \int \cdots \int_{-\infty}^{\infty} \mathbf{\Phi}(\underline{X}) \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{\partial^2}{\partial X_i \partial X_j} \{b_{ij}(\underline{X}, t) p(\underline{X}, t)\}\, \mathrm{d}X_1\, \mathrm{d}X_2 \cdots \mathrm{d}X_n.
\end{aligned} \tag{5}$$

It can be rewritten for second order moment as

$$\frac{\partial}{\partial t} E[X_1^i X_2^j] = \int \int_{-\infty}^{\infty} X_1^i X_2^j \frac{\partial}{\partial t} p(\underline{X}, t)\, \mathrm{d}X_1\, \mathrm{d}X_2. \tag{6}$$

Dynamic moment equations consist of mean (first order moment), mean square (second order moment), PSD value, system parameters, etc. Any variation of parameters due to external random fluctuation may be rewritten in terms of PSD-related constants or functions in dynamic moment equations. System variable in time domain is reformed in terms of moment. These moments and PSD value are useful to understand system behavior in the sense of stochastic language. Therefore, dynamic moment equation provides a general description of system behavior in stochastic domain, that is, it can represent physical characteristics of the dynamic system.

A stochastic model is said to be more realistic than a deterministic model. While a deterministic model in time domain is concerned with the prediction of the definite state of a phenomenon, the stochastic model helps to investigate the inherent unavoidable and unexplained variations observed in almost all physical phenomena. It quantifies these variations and allows a sensitivity analysis of the phenomena that is due to the uncertainties [2,4,10].

2.2. Flutter model

A flexible airfoil in random flutter is adopted as a physical model for investigation. A flutter phenomenon is a dynamic instability occurring in an aircraft in flight, at a speed called the flutter speed, where the elasticity of structure plays an essential part in the instability. There is a typical flutter model under the turbulent flow in (Fig. 1). In general, a wing can be modelled as a beam and its movement is addressed in a mode shape function. The bending mode of the system can be expressed by the mode shape function

$$f(y) = \cosh(\lambda y) - \cos(\lambda y) - k_r[\sinh(\lambda y) - \sin(\lambda y)], \tag{7}$$

where

$$k_r = \frac{\cosh(\lambda y) + \cos(\lambda y)}{\sinh(\lambda y) + \sin(\lambda y)}$$

and the torsion mode shape function follows

$$g(y) = \sin \frac{\pi}{2\ell} y. \tag{8}$$

Using mode shape Eqs. (7) and (8), the kinetic energy of system is

$$E_T = \tfrac{1}{2} M \dot{h}^2 + \tfrac{1}{2} I_\alpha \dot{\alpha}^2 + S_\alpha \dot{h} \dot{\alpha}, \tag{9}$$

where

$$M = \int_0^\ell m(y)[f(y)]^2 \, \mathrm{d}y,$$

$$I_\alpha = \int_0^\ell I_\alpha(y)[g(y)]^2 \, \mathrm{d}y,$$

$$S_\alpha = \int_0^\ell S_\alpha(y) f(y) g(y) \, \mathrm{d}y$$

and potential energy of system is

$$E_U = \tfrac{1}{2} k_h h^2 + \tfrac{1}{2} k_\alpha \alpha^2, \tag{10}$$

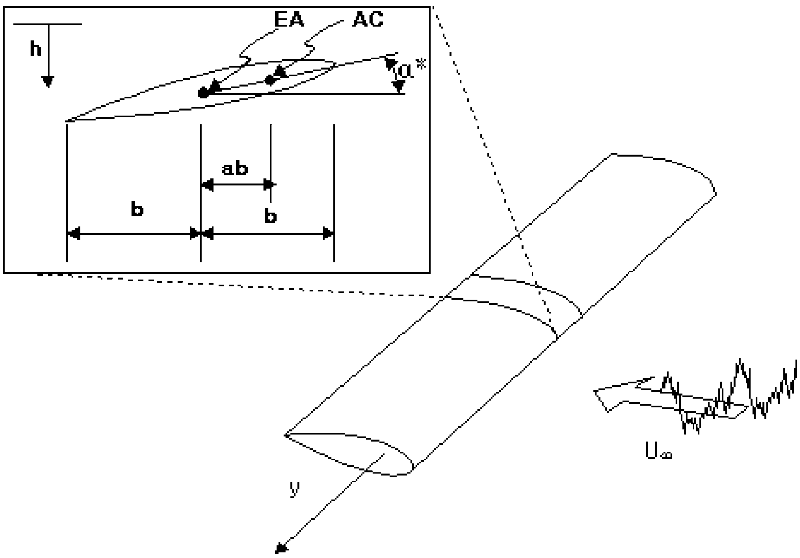

Fig. 1. Flutter model under turbulent flow.

where

$$k_h = \int_0^\ell EI(y)\left\{\frac{\partial^2}{\partial y^2} f(y)\right\} \mathrm{d}y,$$

$$k_\alpha = \int_0^\ell GJ(y)\left\{\frac{\partial}{\partial y} g(y)\right\}^2 \mathrm{d}y.$$

The unsteady aerodynamic load on system consists of lift and moment; unsteady aerodynamic lift L is

$$L = \pi\rho b^2[\ddot{h} + U\dot{\alpha} - ba\ddot{\alpha}] + 2\pi\rho b U C(k)\left[\dot{h} + U\alpha + b\left(\tfrac{1}{2} - a\right)\dot{\alpha}\right] \tag{11}$$

and unsteady aerodynamic moment M_y is

$$\begin{aligned} M_y = {} & \pi\rho b^2\left[ba\ddot{h} + Ub\left(\tfrac{1}{2} - a\right)\dot{\alpha} - b^2\left(\tfrac{1}{8} + a^2\right)\ddot{\alpha}\right] \\ & + 2\pi\rho U b^2\left(a + \tfrac{1}{2}\right)C(k)\left[\dot{h} + U\alpha + b\left(\tfrac{1}{2} - a\right)\dot{\alpha}\right], \end{aligned} \tag{12}$$

where the L and M_y above are aerodynamic lift and moment along the unit length of wing, so total external aerodynamic lift and moment on the system can be evaluated by using mode shape as follows

$$\begin{aligned} Q_h &= A_{hh1}\ddot{h} + A_{hh2}\dot{h} + A_{h\alpha 1}U\ddot{\alpha} + A_{h\alpha 2}U\dot{\alpha} + A_{h\alpha 3}U^2\alpha, \\ Q_\alpha &= A_{\alpha h1}\ddot{h} + A_{\alpha h2}U\dot{h} + A_{\alpha\alpha 1}\ddot{\alpha} + A_{\alpha\alpha 2}U\dot{\alpha} + A_{\alpha\alpha 3}U^2\alpha. \end{aligned} \tag{13}$$

Dynamics of system is summarized as follows:

$$\begin{aligned} & \begin{bmatrix} (M + A_{hh1}) & (S_\alpha + A_{ha1}) \\ (S_\alpha - A_{ah1}) & (I_\alpha - A_{aa1}) \end{bmatrix} \begin{pmatrix} \ddot{h} \\ \ddot{\alpha} \end{pmatrix} + \begin{bmatrix} A_{hh2}\dot{h}U & A_{ha2}U \\ -A_{ah2}U & -A_{aa2}U \end{bmatrix} \begin{pmatrix} \dot{h} \\ \dot{\alpha} \end{pmatrix} \\ & \qquad + \begin{bmatrix} K_h & A_{ha3}U^2 \\ 0 & (K_a - A_{aa3}U^2) \end{bmatrix} \begin{pmatrix} h \\ \alpha \end{pmatrix} = \underline{0}. \end{aligned} \tag{14}$$

If the mass matrix is invertible, then the system can be rewritten as

$$\ddot{\underline{q}} + \underline{\underline{\tilde{D}}}U\dot{\underline{q}} + \underline{\underline{\tilde{K}}}q = 0, \tag{15}$$

where

$$\begin{aligned} \underline{\underline{\tilde{D}}} &= \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix} \\ &= \begin{bmatrix} (M + A_{hh1}) & (S_\alpha + A_{ha1}) \\ (S_\alpha - A_{ah1}) & (I_\alpha - A_{aa1}) \end{bmatrix}^{-1} \begin{bmatrix} A_{hh2} & A_{ha2} \\ -A_{ah2} & -A_{aa2} \end{bmatrix}, \end{aligned}$$

$$\underline{\underline{\tilde{K}}} = \begin{bmatrix} k_{11} & k_{12} + k'_{12}U^2 \\ k_{21} & k_{22} + k'_{22}U^2 \end{bmatrix}$$
$$= \begin{bmatrix} (M + A_{hh1}) & (S_\alpha + A_{ha1}) \\ (S_\alpha - A_{ah1}) & (I_\alpha - A_{aa1}) \end{bmatrix}^{-1} \begin{bmatrix} K_h & A_{ha3}U^2 \\ 0 & (K_a - A_{aa3}U^2) \end{bmatrix}.$$

We assume that the Theodorsen function $C(k)$ is 1, that is, aerodynamic load is a quasi-steady case. Air speed U is defined as

$$U = U_\infty + U_t, \tag{16}$$

where U_t is assumed having white noise characteristics with zero mean and PSD D_t [9]

When F–P–K procedure is applied to Eq. (15), we get the first order moment equations and the second order moment equations as follows

First order dynamic moment equation

$$\begin{aligned}
\dot{m}_{1000} &= m_{0010}, \\
\dot{m}_{0100} &= m_{0001}, \\
m_{0010} &= -k_{11}m_{1000} - (k_{12} + k'_{12}U_\infty^2 + k'_{12}D_t)m_{0100}, \\
m_{0001} &= -k_{21}m_{1000} - (k_{22} + k'_{22}U_\infty^2 + k'_{22}D_t)m_{0100}.
\end{aligned} \tag{17}$$

Second order dynamic moment equation

$$\begin{aligned}
\dot{m}_{2000} &= 2m_{1010}, \\
\dot{m}_{1100} &= m_{0110} + m_{1001}, \\
\dot{m}_{1010} &= m_{0020} - k_{11}m_{2000} - (k_{12} + k'_{12}U_\infty^2 + k'_{12}D_t)m_{1100} - k'_{12}D_t m_{0110}, \\
\dot{m}_{1001} &= m_{0011} - k_{21}m_{2000} - (k_{22} + k'_{22}U_\infty^2 + k'_{22}D_t)m_{1100} - k'_{22}D_t m_{0110},
\end{aligned}$$

$$\begin{aligned}
\dot{m}_{0200} &= 2m_{0101}, \\
\dot{m}_{0110} &= m_{0011} - k_{11}m_{1100} - (k_{12} + k'_{12}U_\infty^2 + k'_{12}D_t)m_{0200} - k'_{12}D_t m_{0101}, \\
\dot{m}_{0101} &= m_{0002} - k_{21}m_{1100} - (k_{22} + k'_{22}U_\infty^2 + k'_{22}D_t)m_{0200} - k'_{22}D_t m_{0101},
\end{aligned}$$

$$\begin{aligned}
\dot{m}_{0020} = {} & 2k_{11}k'_{12}D_t m_{1100} + 2k_{11}m_{1010} + [2(k_{12}k'_{12} + k'^2_{12}U_\infty + 2k'^2_{12}U_\infty^2)D_t + k'^2_{12}D_t^2]m_{0200} \\
& - 2[(k_{12} + k'_{12}U_\infty^2) + (k'_{12} - 3d_{11}k'_{12}U_\infty)D_t]m_{0110} + 6d_{11}k'_{12}U_\infty D_t m_{0101} \\
& + d_{11}^2 D_t m_{0020} + 2d_{11}d_{12}D_t m_{0011} + d_{12}^2 D_t m_{0002},
\end{aligned} \tag{18}$$

$$\begin{aligned}
\dot{m}_{0011} = {} & (k_{11}k'_{22} + k_{21}k'_{12})D_t m_{1100} - k_{21}m_{1010} - k_{11}m_{1001} \\
& + [(k_{12}k'_{22} + 2k'_{12}k'_{22}U_\infty + 4k'_{12}k'_{22}U_\infty^2 + k'_{12}k_{22})D_t + k'_{12}k'_{22}D_t^2]m_{0200} \\
& + [3(d_{21}k'_{12}U_\infty + d_{11}k'_{22}U_\infty - k'_{22})D_t - (k_{22} + k'_{22}U_\infty^2)]m_{0110} \\
& + [3(d_{22}k'_{12}U_\infty + d_{12}k'_{22}U_\infty - k'_{12})D_t - (k_{12} + k'_{12}U_\infty^2)]m_{0101} \\
& + d_{11}d_{21}D_t m_{0020} + (d_{11}d_{22} + d_{12}d_{21})D_t m_{0011} + d_{12}d_{22}D_t m_{0002},
\end{aligned}$$

$$\begin{aligned}\dot{m}_{0002} = {} & 2k_{21}k'_{22}D_t m_{1100} + 2k2_{11}m_{1010} + [2(k_{22}k'_{22} + k'^2_{12}U_\infty + 2k'^2_{22}U^2_\infty)D_t + k'^2_{22}D^2_t]m_{0200} \\ & + 6d_{21}k'_{22}U_\infty D_t m_{0110} - 2[(k_{22} + k'_{22}U^2_\infty) + (k'_{22} - 3d_{21}k'_{22}U_\infty)D_t]m_{0101} \\ & + d^2_{21}D_t m_{0020} + 2d_{21}d_{22}D_t m_{0011} + d^2_{22}D_t m_{0002}.\end{aligned}$$

Eqs. (17) and (18) can be simplified as matrix form

$$\underline{\dot{m}} = \underline{\underline{A}}_m \underline{m}. \tag{19}$$

System matrix $\underline{\underline{A}}_m$ is in deterministic form with constant PSD. As shown in Eq. (15), it is a randomly varying system in time domain, because air speed U varies along time in random manner. However, the random flutter system is transformed to deterministic form, Eq. (19), in stochastic domain. Therefore the random flutter system in time domain can be handled like a deterministic system in stochastic domain [4,10].

3. Controller design in stochastic domain

3.1. Stability in stochastic domain

Relations between system stability in stochastic domain and time domain are important properties to credit the ability of the proposed stochastic controller. If a system in stochastic domain has conditions sufficient for stability, the system in time domain can be stabilized or controlled by using the proposed stochastic controller.

The stability of a system is an important topic in system design and analysis. Before discussing the stability in stochastic domain, stability in time domain may be considered first. The followings are widely well known.

In time domain, a system is stable if all states of the system are bounded. That is to say that

$$|x(t)| \leqslant A \tag{20}$$

for all t where A is some finite constant and $|x(t)|$ denotes the absolute value of $x(t)$. In this case, the system is stable (or marginally stable). If the state approaches zero as t becomes large, such systems are considered to be asymptotically stable. These stability definitions can also be stated in terms of eigenvalues of the system or in terms of the poles of the transfer function of the system. The system is marginally stable if the poles or eigenvalues of system lie along the imaginary axis, unstable if one or more poles or eigenvalues lie in the right half-plane, and asymptotically stable if all of the poles or eigenvalues lie in the left half-plane.

A system may be described in terms of dynamic moment in stochastic domain, and dynamic moment equations can be rewritten in form of state space equations in time domain. Therefore, system stability in stochastic domain can be determined in the same manner as in time domain.

As an example, a one-degree-of-freedom dynamic system is considered. Typical one-degree of freedom dynamic system under parametric random disturbance and control describe as

$$\ddot{y} + 2\zeta\omega\dot{y} + \omega^2 y = f(\dot{y}, y), \tag{21}$$

where

$$f(\dot{y}, y) = f_d(\dot{y}, y) + f_c(\dot{y}, y)$$

$f_d(\dot{y}, y)$ is the external random disturbance and $f_c(\dot{y}, y)$ the control input.

It can be described as a stability problem of parametric system in stochastic sense.

$$\ddot{y} + 2\zeta\omega(1 + g_\zeta)\dot{y} + \omega^2(1 + g_k)y = 0, \tag{22}$$

where g_k is white noise type random fluctuation of stiffness having power spectral density D_{kk} and g_ζ is white noise type random fluctuation of damping having power spectral density $D_{\zeta\zeta}$.

Introducing the co-ordinates transformation Eq. (23), Eq. (22) can be written in the Ito's stochastic differential equation (24).

$$\begin{aligned} y &= X_1, \\ \dot{y} &= X_2, \end{aligned} \tag{23}$$

$$\begin{aligned} \mathrm{d}X_1 &= X_2\,\mathrm{d}\tau, \\ \mathrm{d}X_2 &= \{-\omega^2(1 + g_k)X_1 - 2\zeta\omega(1 + g_\zeta)X_2\}\,\mathrm{d}\tau. \end{aligned} \tag{24}$$

The evolution of transitional joint probability density function of the response co-ordinates $P(\underline{X}, \tau)$ can be described by the F–P–K equation.

$$\frac{\partial}{\partial\tau}P(\underline{X}, \tau) = -\sum_{i=1}^{2}\frac{\partial}{\partial X_i}[a_i(\underline{X}, \tau)P(\underline{X}, \tau)] + \frac{1}{2}\sum_{i=1}^{2}\sum_{j=1}^{2}\frac{\partial^2}{\partial X_i \partial X_j}[b_{ij}(\underline{X}, \tau)P(\underline{X}, \tau)], \tag{25}$$

where $a_i(\underline{X}, \tau)$ is the 1st incremental moment or drift coefficient, $b_{ij}(\underline{X}, \tau)$ the second incremental moment or diffusion coefficient.

The general differential equation for moments is

$$\frac{\partial}{\partial\tau}E[X_1^i X_2^j] = \int\int_{-\infty}^{\infty} X_1^i X_2^j \frac{\partial}{\partial\tau}P(\underline{X}, \tau)\,\mathrm{d}X_1\,\mathrm{d}X_2. \tag{26}$$

Thus the first order moment equations and the second order moment equations are obtained as Eq. (27).

$$\begin{aligned} \dot{m}_{10} &= m_{01}, \\ \dot{m}_{01} &= -2\zeta\omega m_{01} - \omega^2 m_{10}, \\ \dot{m}_{11} &= m_{02} - 2\zeta\omega m_{11} - \omega^2 m_{20} + 4\zeta\omega^3 D_{k\zeta} m_{11}, \\ \dot{m}_{20} &= 2m_{11} + 2\zeta\omega^4 D_{kk} m_{20}, \\ \dot{m}_{02} &= -4\zeta\omega m_{02} - 2\omega^2 m_{11} + 2\omega^4 D_{\zeta\zeta} m_{02}. \end{aligned} \tag{27}$$

The influence on the stability of the system due to the degree of random fluctuation of damping and stiffness is examined along the variation of power spectral densities $(D_{kk}, D_{\zeta\zeta}, D_{k\zeta})$. The stable and unstable regions are shown in Figs. 2 and 3, respectively.

Responses of dynamic moment in stable region (Lower space: Zone 1) and unstable region (Upper space: Zone 2) are shown in Figs. 4 and 5.

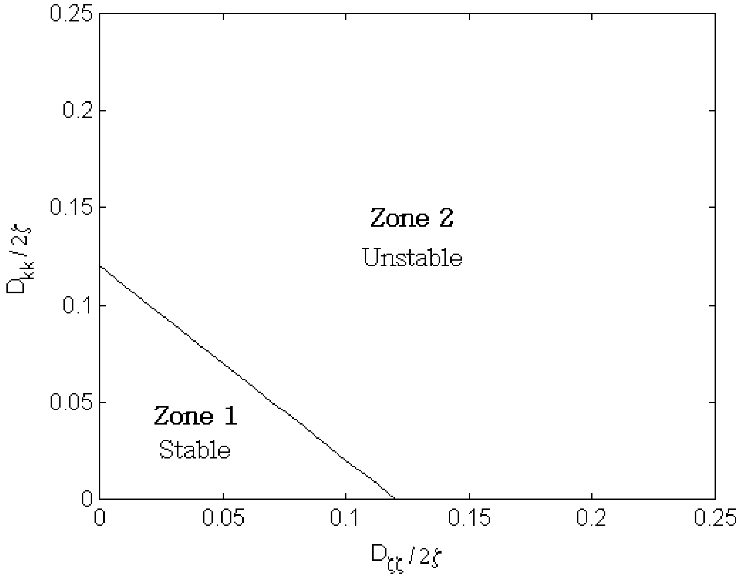

Fig. 2. Stability region on P.S.D plane.

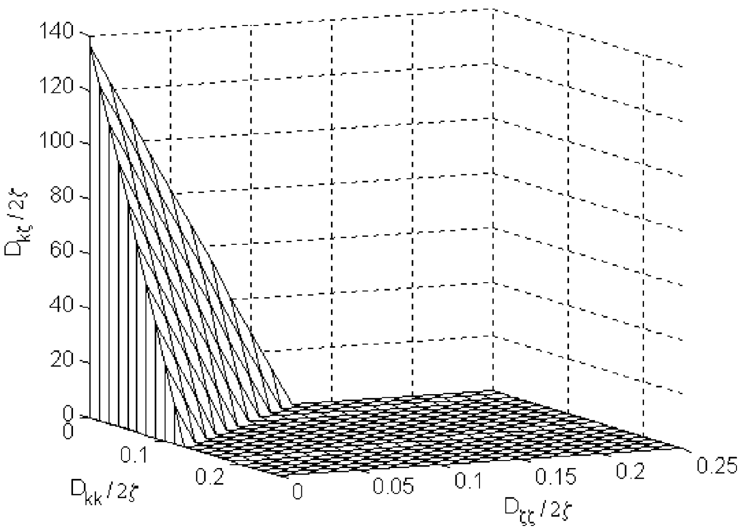

Fig. 3. Stability region on P.S.D space.

According to "Stability Analysis of a Dynamic System under Random Parametric Excitation" by Heo et al. in 1997, if a system in the stochastic domain is stable, then it is also stable in time domain. Therefore, dynamic system in time domain can be controlled or stabilized by the control of dynamic moment in stochastic domain [7,10].

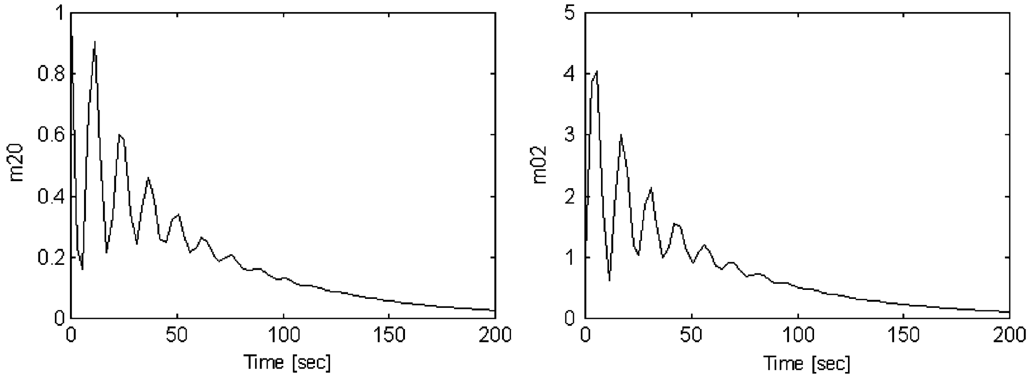

Fig. 4. Stable moment response of the system (Zone 1).

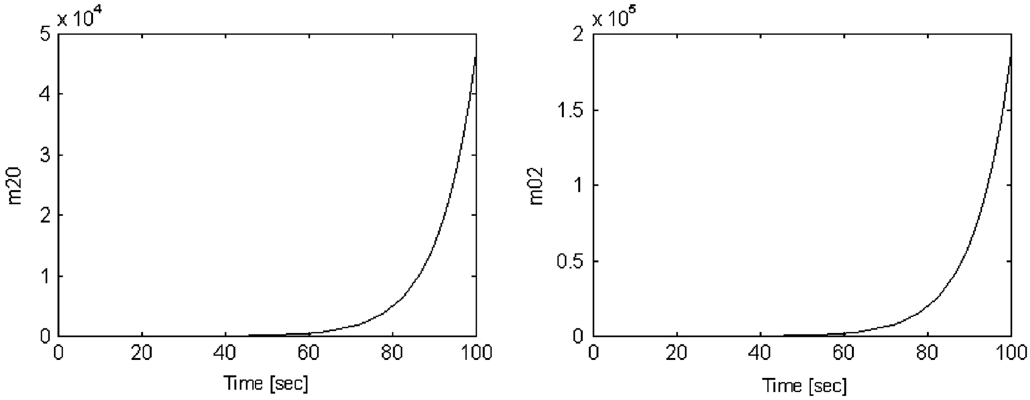

Fig. 5. Unstable moment response of the system (Zone 2).

3.2. Proposed stochastic controller

Fig. 6 is a schematic diagram that shows the concept of the proposed stochastic controller.

The proposed stochastic controller can be designed as follows. First, system dynamics may be derived in time domain, and then it is transformed to dynamic moment equations in stochastic domain by F–P–K procedure. Next, the controller can be designed in stochastic domain by using dynamic moment equations. In designing a controller in stochastic domain, most of controller design techniques used in time domain can be applied. The object of controller design here is to reduce the dynamic moment response. Finally, using the obtained PSD in stochastic domain, the control signal can be generated by using Monte-Carlo method in physical time domain [4,10].

When control force is applied to the system in flutter that is mentioned above, Eq. (17) and Eq. (18) can be rewritten as matrix form

$$\underline{\dot{m}} = \underline{\underline{D}}_m \underline{m} + \underline{B}_m D_C. \tag{28}$$

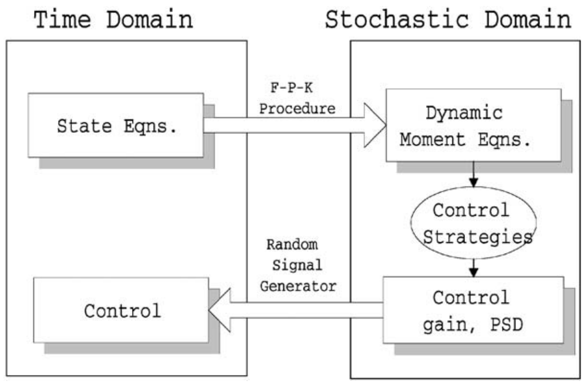

Fig. 6. Conceptual diagram of proposed stochastic controller.

Control gain matrix can be modelled through the F–P–K procedure. Control PSD value can be obtained by applying Eq. (28) to any general control design procedure in time domain.

3.3. System modelling

A general flutter model for a wing has already been introduced and its F–P–K equation is derived in the previous section. Using the strip theory in aerodynamic modelling, the system equation for an aeroelasticity model under control can have the following form

$$M\ddot{h} + c_s\dot{h} + k_h h = -L + F, \tag{29}$$

where

$$M = \int_0^{\ell} m(y)[f(y)]^2\,\mathrm{d}y,$$

$$k_h = \int_0^{\ell} EI(y)\left\{\frac{\partial^2}{\partial y^2}f(y)\right\}^2\mathrm{d}y.$$

Unsteady aerodynamic lift can be rewritten as

$$L = A_{hh1}\ddot{h} + A_{hh2}U\dot{h}, \tag{30}$$

where

$$A_{hh1} = \int_0^{\ell} \pi\rho b^2[f(y)]^2\,\mathrm{d}y,$$

$$A_{hh2} = \int_0^{\ell} 2\pi\rho bC(k)[f(y)]^2\,\mathrm{d}y.$$

For the purpose of demonstrating the proposed controller, only one bending mode is considered here, and then from Eq. (29) and Eq. (30) a system dynamic follows [9]

$$\ddot{h} + (\tilde{C}_1 + U_t\tilde{C}_2)\dot{h} + \tilde{K}h = \tilde{F}, \tag{31}$$

where $C(k) = 1$

$$\tilde{C}_1 = \frac{c_s + U_\infty A_{hh2}}{(M + A_{hh1})}, \quad \tilde{C}_2 = \frac{A_{hh2}}{(M + A_{hh1})}, \quad \tilde{K} = \frac{k_h}{(M + A_{hh1})}, \quad \tilde{F} = \frac{F}{(M + A_{hh1})}.$$

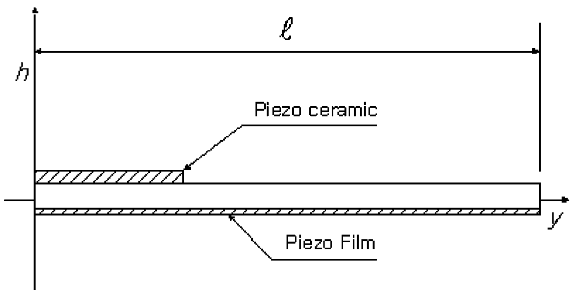

Fig. 7. Structure of airfoil model in control.

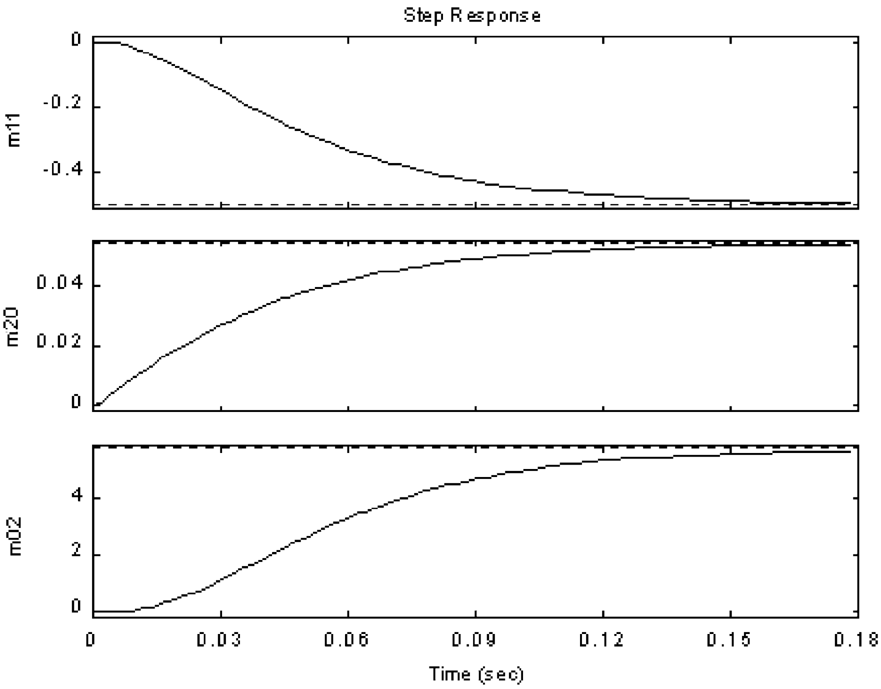

Fig. 8. Dynamic moment response on step input ($PSD = 0$).

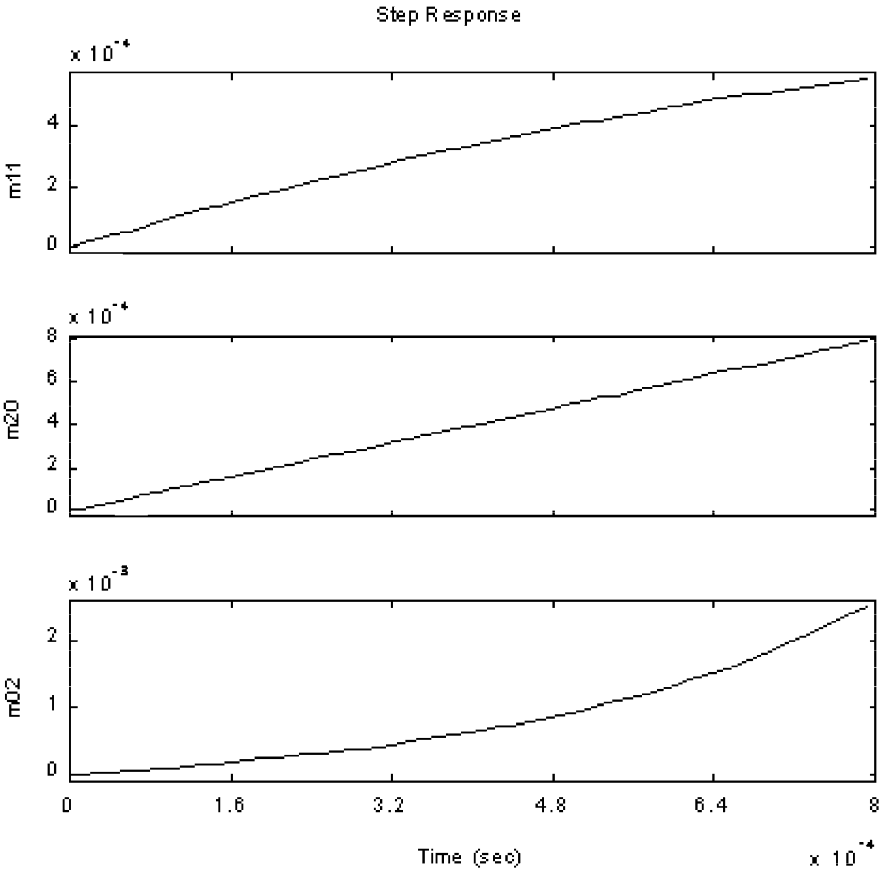

Fig. 9. Dynamic moment response on step input ($PSD = 894$).

Piezo ceramic is modelled to be placed on the root section as an actuator and piezo film is also modelled to be placed on one side of the airfoil as a sensor, respectively. The schematic diagram of airfoil with an actuator and a sensor is shown in (Fig. 7). Also control force is defined as

$$F(t) = C_p V(t)\delta'(x - \ell) = \tilde{B}V(t), \tag{32}$$

where the coefficient $\tilde{B}$ does not depend on time but the geometric co-ordinate of the beam [4]. Aeroelastic structure is assumed to be exposed to turbulent flow having white noise type random fluctuation component with power spectral density D_t. Control voltage $V(t)$ also satisfies the condition of white noise type random fluctuation and its PSD value is D_V. As shown in Eq. (31), one bending mode flutter model is formed to time-varying system in a random manner, which is a random parametric system.

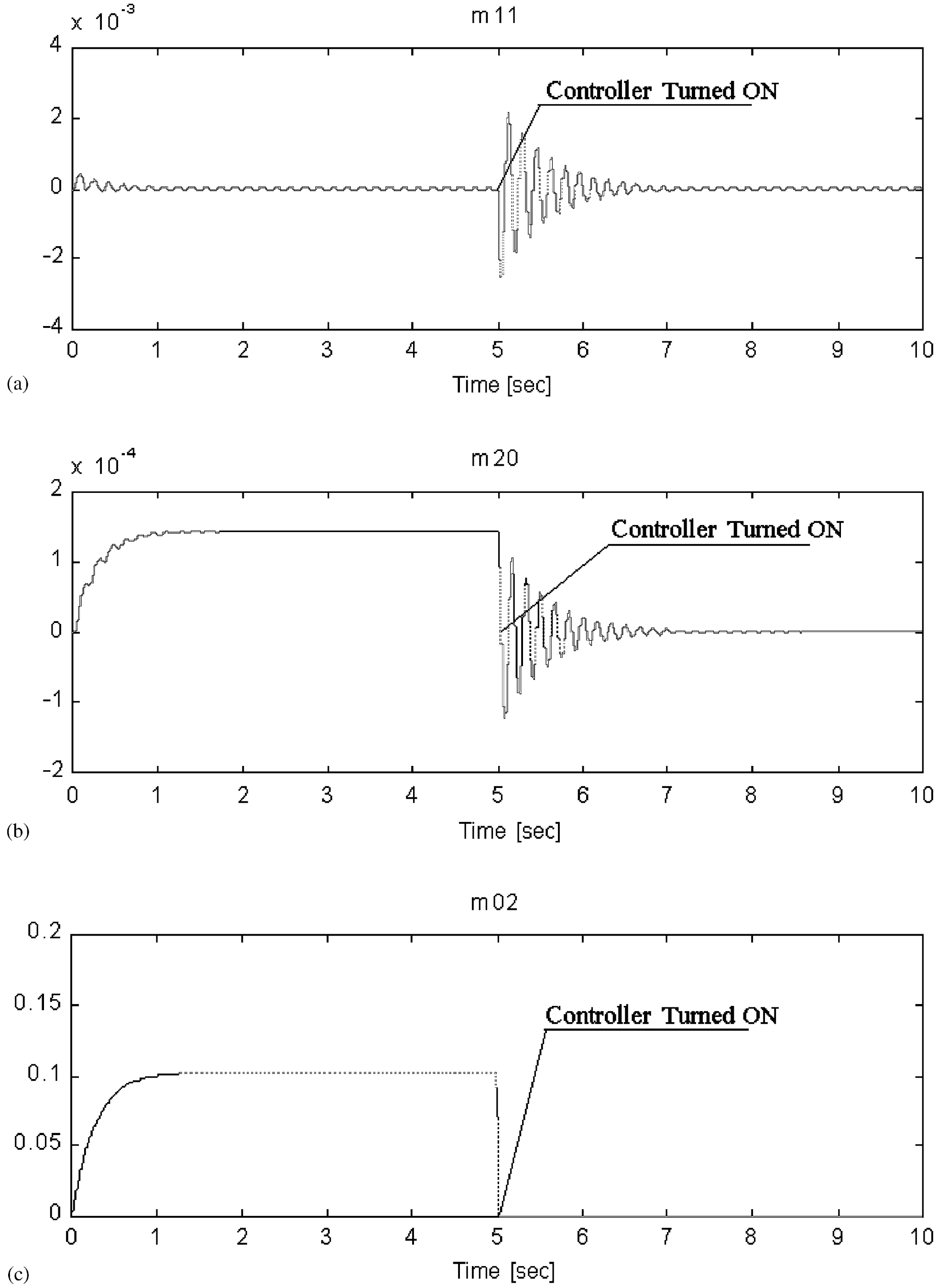

Fig. 10. Dynamic moment response under control.

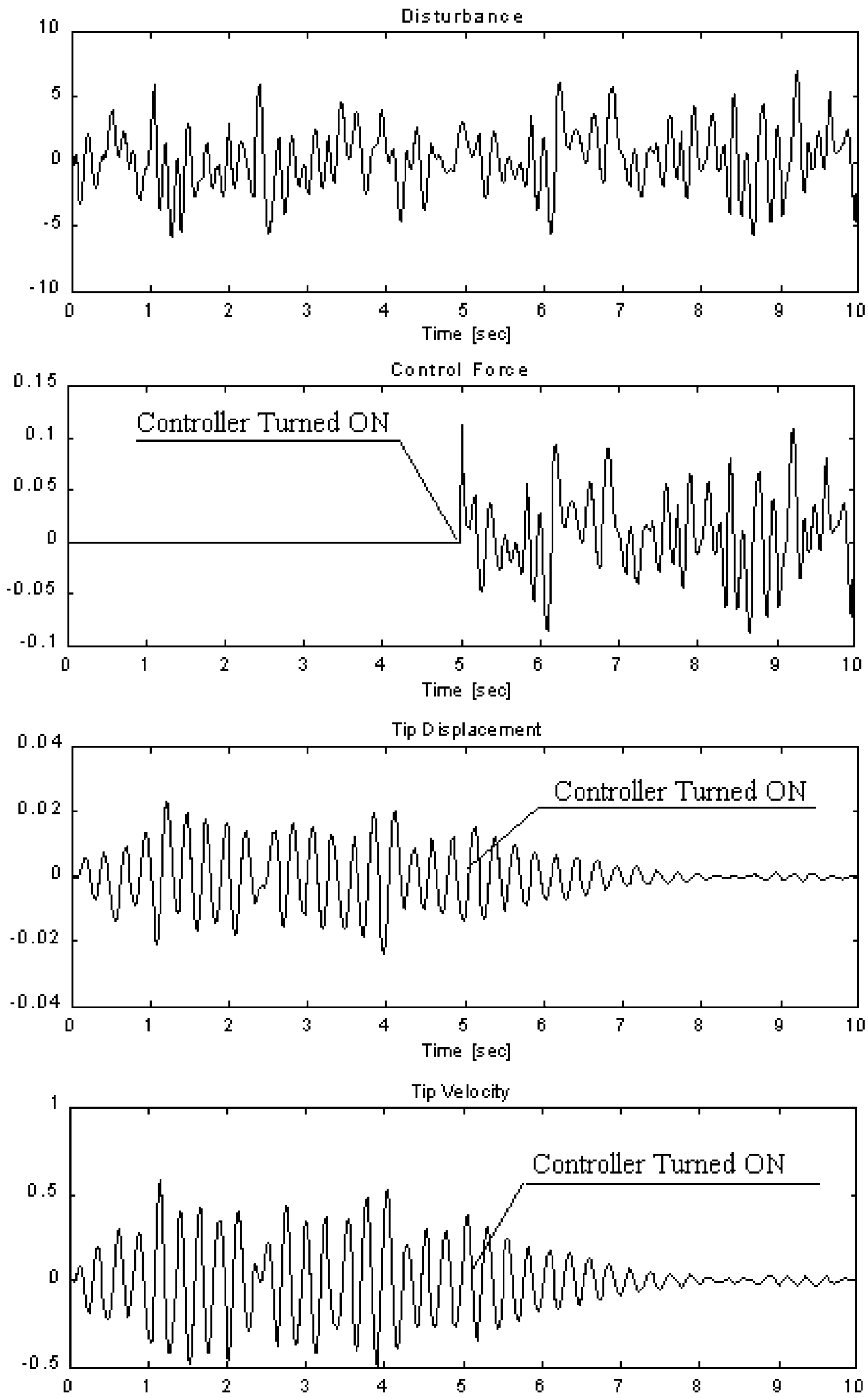

Fig. 11. Physical response under control.

When the F–P–K procedure is applied to Eq. (31), the first order moment equations and the second order moment equations are obtained as follows

$$\begin{aligned}
\dot{m}_{10} &= m_{01}, \\
\dot{m}_{01} &= -\tilde{K}m_{10} - \tilde{C}_1\omega m_{01}, \\
\dot{m}_{11} &= -\tilde{C}_1 m_{11} + m_{02} - \tilde{K}m_{20}, \\
\dot{m}_{20} &= 2m_{11}, \\
\dot{m}_{02} &= -2\tilde{K}m_{11} - (2\tilde{C}_1 - \tilde{C}_2 D_t)m_{02} + \tilde{B}^2 D_V.
\end{aligned} \tag{33}$$

Since first order moments of dynamic moment equation are converse to zero in steady state cases, controller design is focused in second order moment equations [4,8,10].

$$\underline{\dot{m}} = \underline{\underline{A}}\underline{m} + \underline{B}u, \tag{34}$$

where system matrix:

$$[A] = \begin{bmatrix} -\tilde{C}_1 & -\tilde{K} & 1 \\ 2 & 0 & 0 \\ -2\tilde{K} & 0 & -2\tilde{C}_1 + \tilde{C}_2 D_t \end{bmatrix}.$$

Control gain matrix:

$$[B] = \begin{bmatrix} 0 \\ 0 \\ \tilde{B}^2 \end{bmatrix}.$$

Control signal: $u = D_V$

4. Simulation and result

The flutter speed of the airfoil under clean air from U-g method is 16 m/s. For the sake of verifying the controller effect, the mean air velocity for turbulent flow-induced flutter is set to 15.5 m/s, and the numerical value of PSD of turbulent flow is 894. The controller is designed in stochastic domain to minimize the dynamic moment response so the airfoil will be stable under flutter in turbulent flow induced disturbance and the PI method is utilized as a control strategy in the study.

Fig. 8 shows uncontrolled dynamic moment response when the airfoil is exposed to clean airflow without turbulence. When the airfoil is under flutter in turbulent flow, step responses of the unstable system in terms of dynamic moments are shown in Fig. 9.

Fig. 10 shows dynamic moment response under control. As shown in the figures, the controller was turned on at 5 s. Time response of the system is shown in Fig. 11, in the same manner as in dynamic moment response, the controller was turned on at 5 s after the airfoil encountered flutter. Velocity and displacement response of the system was reduced about by 10 times. ‘‘Heo-PI controller’’ shows good performance in flutter suppression even though the system experiences very high level of disturbance. Also the performance of the proposed stochastic controller in

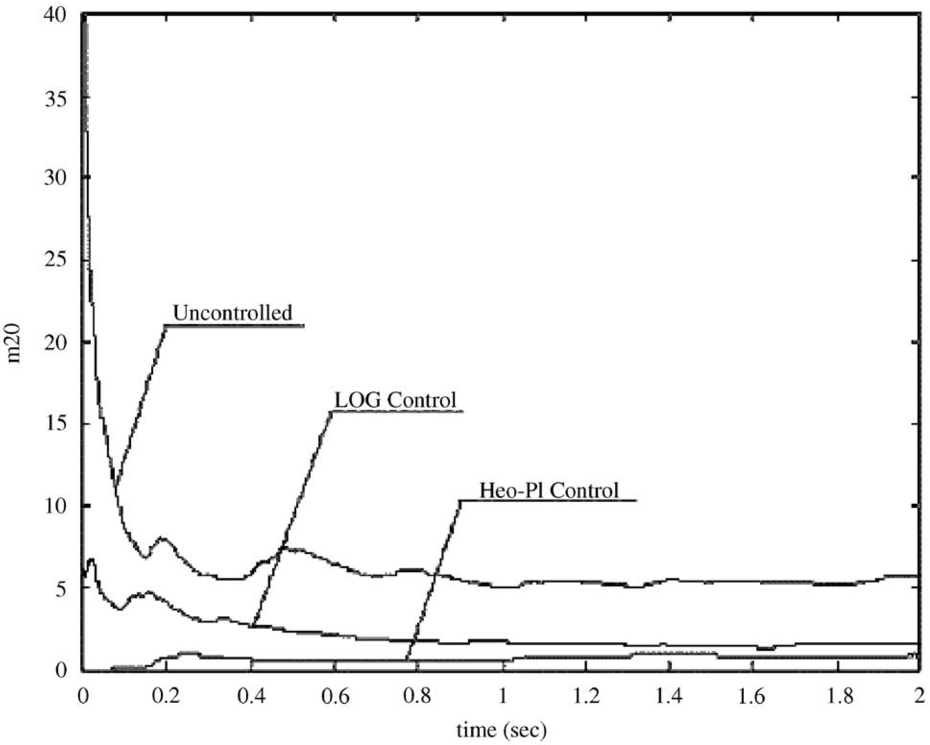

Fig. 12. Comparison of mean square response of an airfoil under flutter.

physical experiment is compared with that of well-known conventional controller, LQG, in Fig. 12 [11].

5. Conclusion

Most of dynamic system may be disturbed in random manner internally or externally. Up to now, many controllers for stochastic dynamic systems have been designed in time domain or frequency domain. However, a newly proposed methodology designed in stochastic domain is introduced and applied successfully to a flexible airfoil in random flutter.

In the system, the PSD value for the control signal obtained in stochastic domain is realized to time series by the Monte-Carlo method in physical domain. Accordingly, the physical system is controlled by random type control force in time domain. Controller gains are tuned along with random disturbance and system parameters as well. It improves the system ability to suppress random disturbances.

The major merit of the ‘‘Heo stochastic control method’’ is that it easily deals with the systems random characteristics, which can be described as a constant or simple function in terms of PSD. Although the controller mentioned here is designed in a simple classical way in stochastic domain, the performance of the proposed ‘‘Heo stochastic controller’’ seems to open new horizon in the area of stochastic control. In stochastic domain, controllers can be designed using already developed controllers such as PID etc. Other types of combined controllers can be designed according to the system conditions or environment. As shown the proposed stochastic controller reveals remarkable performance compared to that of the conventional one. Much of the study is on the way to figure out more detailed characteristics of the proposed new “Heo stochastic controller”.

Acknowledgements

This work was supported by grant No.KOSEF-96-0200-07-01-3 from the Basic Research program of the KOSEF (Korea Scientific and Engineering Foundation).

Appendix A. Nomenclature

α^*	angle of attack
$\alpha(t)$	torsional displacement
ζ	damping ratio
$a = \frac{e}{b}$	location of elastic axis measured from aerodynamic center
$\underline{\underline{A}}_m$	system matrix in stochastic domain
$\overline{AC}$	aerodynamic center
$a_i(\underline{X}, \tau)$	drift coefficient
$b_{ij}(\underline{X}, \tau)$	diffusion coefficient
b	semi chord of airfoil
$\underline{B}_m$	control gain matrix in stochastic domain
$\tilde{B}$	modal control force coefficient for bending motion in airflow
$C(k)$	Theodorsen function
c_s	structural damping
$\tilde{C}_i$	ith modal bending damping coefficient in airflow
C_p	piezo-ceramic coefficient
D_t	power spectral density of turbulent airflow
D_c	power spectral density of control signal
D_v	power spectral density of control voltage
$\underline{\underline{\tilde{D}}}$	modal damping matrix of wing in airflow
$D_{\zeta\zeta}$	auto power spectral density of damping fluctuation
D_{kk}	auto power spectral density of stiffness fluctuation
$D_{k\zeta}$	cross-power spectral density of stiffness fluctuation and damping fluctuation
$E[.]$	expectation operator
EA	elastic axis
EI	elastic axis
E_T	kinetic energy of wing
E_U	potential energy of wing
e	distance between AC and EA
$f(y)$	bending mode shape function

$f_d(\dot{y}, y)$	external random disturbance
$f_c(\dot{y}, y)$	control input
F	control force
$\tilde{F}$	modal control force for bending motion in airflow
GJ	torsion stiffness
$g(y)$	torsional mode shape function
g_k	white noise type random fluctuation of stiffness
g_ζ	white noise type random fluctuation of damping
$h(t)$	bending displacement
I_α	mass moment of inertia about elastic axis of wing
$\underline{\underline{\tilde{K}}}$	modal stiffness matrix of wing in airflow
$\tilde{K}$	modal bending stiffness matrix of wing in airflow
k_h	bending stiffness of wing
k_α	torsional stiffness of wing
$k = \frac{\omega b}{U}$	reduced frequency
ℓ	half-span of wing
L	aerodynamic lift
M_y	aerodynamic moment
M	mass matrix of wing
$m_{ij} = E[X_1^i X_2^j]$	moment
$\underline{m}$	3×1 Dynamic moment vectors
$P(\underline{X}, t)$	transitional joint probability density function
q	modal co-ordinate
S_α	static unbalance of wing
U	air speed
U_∞	mean air speed
U_t	component of turbulent airflow
$V(t)$	control voltage
y	span wise co-ordinate of wing
ω	frequency
ρ	air density
δ	Dirac delta function
δ'	derivative of Dirac delta function

References

[1] R.A. Ibrahim, Parametric Random Vibration, Research Studies Press Letchworth, England, 1985.

[2] Richard Y. Chiang, Michael G. Safonov, Math Works, Robust Control Toolbox For Use with MATLAB, 1992.

[3] Minsung Kim, Jungyoup Han, Hoon Heo, A New Approach to Stochastic Control of Randomly Disturbed System, International Mechatronics Conference, Vol. I, Chejudo, 1–3 December 1995, pp. 440–443.

[4] Yun-Hyun Cho, Korea University M.S. Thesis, A Study of Stochastic Controller Design for a Dynamic System under Irregular Disturbance and Realizing Technique, 1998.
[5] A. Papoulis, Probability, Random Variables, and Stochastic Processes, McGraw-Hill, New York, 1991.
[6] R.A. Ibrahim, H. Heo, Stochastic response of nonlinear structures with parameter random fluctuation, American Institute of Aeronautics and Astronautics Journal 25 (2) (1987) 331–338.
[7] H. Heo, Y.H. Cho, JaeHyuk Yang, Stability Analysis of a Dynamic System under Random Parametric Excitation, K.S.N.V.E Spring Conference, 1997, pp. 55–57.
[8] Y.H. Cho, JaeHyuk Yang, D.J.Kim, H. Heo, Control of the flexible system under irregular disturbance by using of 'Random gain', ICASE Conference, 1998, pp. 435–439.
[9] H.E. Dowell, E.F. Crawley, H.C. Curtiss Jr., D.A. Peters, R.H. Scanlan, Fernando Sisto, A Modern Course in Aero Elasticity, Kluwer Academic Publishers, Dordrecht, 1995.
[10] H. Heo, J. Han, D.J. Kim, Y.-H. Cho, Design Method of Controller in Stochastic Domain, 2000, No. 0327508, patent in Korea.
[11] D.J. Kim, S.T. Park, J.W. Chung, H. Heo, An experimental study on the stochastic control of an aero-elastic system, K.S.M.E. Journal A 23 (11) (1999) 2007–2013.